Problem 1. Solve $2x^2 + 11x + 5 = 0$ by factoring.

Solution. $(2x+1)(x+5) = 0$, and so $x = -\frac{1}{2}, -5$. □

Problem 2. State the quadratic formula and copy the proof from page 49 of the textbook.

Solution. The roots of $ax^2 + bx + c = 0$ $(a \neq 0)$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

See page 49 for a proof. □

Problem 3. Solve $2x^2 + 11x + 5 = 0$ by completing the squares.

Solution.

$$\begin{aligned} 2x^2 + 11x + 5 &= 0 \\ x^2 + \frac{11}{2}x + \frac{5}{2} &= 0 \\ x^2 + \frac{11}{2}x + \frac{121}{16} - \frac{121}{16} + \frac{5}{2} &= 0 \\ \left(x + \frac{11}{4}\right)^2 &= \frac{121}{16} - \frac{5}{2} \\ \left(x + \frac{11}{4}\right)^2 &= \frac{81}{16} \\ x + \frac{11}{4} &= \pm\frac{9}{4} \\ x &= -\frac{1}{2}, -5. \end{aligned}$$

□

Problem 4. Solve $2x^2 + 11x + 5 = 0$ using the quadratic formula.

Solution.

$$x = \frac{-11 \pm \sqrt{121 - 40}}{4} = \frac{-11 \pm 9}{4} = -\frac{1}{2}, -5.$$

□

Problem 5. What quantity of pure acid must be added to 500mL of a 50% acid solution to produce a 60% acid solution?

Solution. Note that the original solution contains

$$500 \text{ mL } \times 50\% = 250 \text{ mL}$$

of acid. It thus suffices to solve

$$\frac{250 + x}{500 + x} = 60\% = \frac{6}{10}.$$

Cross-multiplication yields $2500 + 10x = 3000 + 6x$, whereby

$$x = 125.$$

It follows that 125 mL of acid must be added. □

Problem 6. Henry and Irene working together can wash all the windows of their house in 1 h 48 min. Working alone, it takes Henry 1.5 h more than Irene to do the job. How long does it take each person working alone to wash all the windows?

Solution. Let H represent how long it takes Henry to wash all the windows, and I how long it takes Irene to do the job. Then we have

$$\begin{aligned} \frac{1.8}{H} + \frac{1.8}{I} &= 1 \qquad (1) \\ H &= 1.5 + I, \qquad (2) \end{aligned}$$

for 1 h 48 min = 1.8 hr. Plugging (2) into (1) yields

$$\frac{1.8}{1.5 + I} + \frac{1.8}{I} = 1,$$

which is equivalent to

$$\frac{1.8}{1.5 + I} + \frac{1.8}{I} - 1 = 0.$$

Simplifying the expression, we get

$$\frac{-I^2 + 2.1I + 2.7}{I(1.5 + I)} = 0.$$

It thus suffices to solve

$$-I^2 + 2.1I + 2.7 = 0.$$

Multiplying by -10, we obtain

$$10I^2 - 21I - 27 = 0,$$

whose factored form is

$$(10I + 9)(I - 3) = 0.$$

Since $I > 0$, we see that $I = 3$, whence $H = 4.5$. □

Problem 7. A salesman drives from Ajax to Barrington, a distance of 120mi, at a steady speed. He then increases his speed by 10mi/h to drive the 150mi from Barrington to Collins. If the second leg of his trip took 6 min more time than the first leg, how fast was he driving between Ajax and Barrington?

Solution. We suppose that the salesman drove from Ajax to Barrington at x miles per hour. Then he drove from Barrington to Collins at $x + 10$ miles per hour, and so

$$\frac{120}{x} + \frac{1}{10} = \frac{150}{x + 10}.$$

The above equation is equivalent to

$$\frac{120}{x} + \frac{1}{10} - \frac{150}{x+10} = 0,$$

and mighty Algebra yields

$$\frac{x^2 - 290x + 12000}{10x(x+10)} = 0.$$

It thus suffices to solve

$$x^2 - 290x + 12000 = 0,$$

whose factored form is

$$(x - 50)(x - 240) = 0.$$

Both $x = 50$ and $x = 240$ are positive, and so they are mathematically correct—both will therefore be considered correct. Nevertheless, a moment's thought should reveal that 240 miles per hour is just too fast. □

Problem 8. Two fishing boats depart a harbor at the same time, one traveling east, the other south. The eastbound boat travels at a speed 3mi/h faster than the southbound boat. After two hours the boats are 30 mi apart. Find the speed of the southbound boat.

Proof. Let x mi/h be the speed of the southbound boat, so that $x+3$ mi/h is the speed of eastbound boat. After two hours, the eastbound boat will have traveled $2(x + 3)$ miles and the southbound boat $2x$ miles. By the Pythagorean theorem, we have

$$[2(x + 3)]^2 + (2x)^2 = 30^2.$$

Simplifying the above equation yields

$$x^2 + 3x - 108 = 0,$$

whose factored form is

$$(x - 9)(x + 12) = 0.$$

Since $x > 0$, it follows that $x = 9$ mi/h. □