

Problem 1 (5 points). Find the equation of the straight line that is perpendicular to the straight line $3x - 5y = 7$ and passes through the point $(2, 3)$. Write the resulting equation in *slope-intercept form*.

Solution. We first rewrite $3x - 5y = 7$ in slope-intercept form:

$$y = \frac{3}{5}x - \frac{7}{5}.$$

The new slope a must satisfy the equation

$$\frac{3}{5} \cdot a = -1,$$

whence $a = -5/3$. Since the desired line passes through $(2, 3)$, we plug in $a = -5/3$, $x = 2$, and $y = 3$ into $y = ax + b$ to obtain

$$3 = \left(-\frac{5}{3}\right)(2) + b.$$

It follows that $b = 19/3$, and so the desired equation is

$$y = -\frac{5}{3}x + \frac{19}{3}.$$

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Problem 2 (5 points). Solve the following inequality:

$$3x^2 + 5x - 2 < 0.$$

Solution. We first factor the left-hand side as follows:

$$(3x - 1)(x + 2) < 0.$$

The “critical points” are $x = 1/3$ and $x = -2$. If $x < -2$, then $x + 2 < 0$ and $3x - 1 < 0$, and so $(3x - 1)(x + 2) > 0$. If $-2 < x < 1/3$, then $x + 2 > 0$ and $3x - 1 < 0$, and so $(3x - 1)(x + 2) < 0$. If $x > 1/3$, then $x + 2 > 0$ and $3x - 1 > 0$, and so $(3x - 1)(x + 2) > 0$. It follows that

$$-2 < x < \frac{1}{3}.$$

□