

Problem 1 (1 point). Solve $2^{x^2-5} = 2^{x+2}$.

Solution. Since the bases are the same, it suffices to solve $x^2 - 5 = x + 2$. This equation is equivalent to

$$x^2 - x - 7 = 0,$$

whose roots are

$$x = \frac{1 \pm \sqrt{29}}{2}.$$

□

Problem 2 (2 points). Solve $3^{x^2-5} = 27^{x+2}$.

Solution. $27 = 3^3$, so the above equation is equivalent to

$$3^{x^2-5} = 3^{3x+6}.$$

It suffices to solve $x^2 - 5 = 3x + 6$, which is equivalent to

$$x^2 - 3x - 11 = 0.$$

The roots are

$$x = \frac{3 \pm \sqrt{53}}{2}.$$

□

Problem 3 (2 points). Solve $2^{x^2-5} = 8^{x+2}$.

Solution. $27 = 2^3$, so the above equation is equivalent to

$$2^{x^2-5} = 2^{3x+6}.$$

It suffices to solve $x^2 - 5 = 3x + 6$, which is equivalent to

$$x^2 - 3x - 11 = 0.$$

The roots are

$$x = \frac{3 \pm \sqrt{53}}{2}.$$

□

Problem 4 (3 points). Solve $4^{x^2-5} = 8^{x+2}$.

Solution. $4 = 2^2$ and $8 = 2^3$, and so the above equation is equivalent to

$$2^{2x^2-10} = 2^{3x+6}.$$

It suffices to solve $2x^2 - 10 = 3x + 6$, which is equivalent to

$$2x^2 - 3x - 16 = 0.$$

The roots are

$$x = \frac{3 \pm \sqrt{137}}{4}.$$

□