1 Solutions

1.3.15 Expand the first set of parentheses:

$$4 - 10t + t^2(t-1) - (t^4 - 1)$$

Expand the second set of parentheses:

$$4 - 10t + t^3 - t^2 - (t^4 - 1)$$

Expand the third set of parentheses:

$$4 - 10t + t^3 - t^2 - t^4 - 1$$

Collect like terms:

$$-t^4 + t^3 - t^2 - 10t + (4 - 1)$$

Simplify:

$$-t^4 + t^3 - t^2 - 10t + 3$$

1.3.21 Expand:

$$x \cdot 3x + x \cdot -y + 2y \cdot 3x + 2y \cdot -y$$

Simplify each term:

$$3x^2 - xy + 6xy - 2y^2$$

Collect like terms:

$$3x^2 + (-1+6)xy - 2y^2$$

Simplify:

$$3x^2 + 5xy - 2y^2$$

1.3.24 Let $A = 3x$ and $B = 4$:

$$(A+B)^2$$

Use the formula $(A+B)^2 = A^2 + 2AB + B^2$:

$$A^2 + 2AB + B^2$$

Substitute $A = 3x$ and $B = 4$:

$$(3x)^2 + 2(3x)(4) + (4)^2$$

Simplify:

$$9x^2 + 24x + 16$$

1.3.29 Let $A = x^2$ and $B = a^2$

$$(A-B)(A+B)$$

Use the formula $(A-B)(A+B) = (A+B)(A-B) = A^2 - B^2$:

$$A^2 - B^2$$

Substitute $A = x^2$ and $B = a^2$:

$$(x^2)^2 - (a^2)^2$$

Simplify:

$$x^4 - a^4$$

Note: One *could* FOIL it out; it would be a tad tedious, however.

1.3.30 Let $A = x^{1/2}$ and $B = y^{1/2}$

$$(A + B)(A - B)$$

Use the formula $(A + B)(A - B) = A^2 - B^2$:

$$A^2 - B^2$$

Substitute $A = x^{1/2}$ and $B = y^{1/2}$:

$$(x^{1/2})^2 - (y^{1/2})^2$$

Simplify:

$$x - y$$

1.3.31 Let $A = \sqrt{a}$ and $B = \frac{1}{b}$:

$$(A - B)(A + B)$$

Use the formula $(A - B)(A + B) = (A + B)(A - B) = A^2 - B^2$:

$$A^2 - B^2$$

Substitute $A = \sqrt{A}$ and $B = \frac{1}{b}$:

$$(\sqrt{a})^2 - \left(\frac{1}{b}\right)$$

Simplify:

$$a - \frac{1}{b^2}$$

1.3.32 Let $A = \sqrt{h^2 + 1}$ and $B = 1$:

$$(A + B)(A - B)$$

Use the formula $(A + B)(A - B) = A^2 - B^2$

$$A^2 - B^2$$

Substitute $A = \sqrt{h^2 + 1}$ and $B = 1$:

$$(\sqrt{h^2 + 1})^2 - (1)^2$$

Simplify:

$$h^2 + 1 - 1 = h^2$$

1.3.75 $2x^2 + 5x + 3 = (2x + 3)(x + 1)$ (*Magic!*)

1.3.86 Let $A = a^2 - 1$:

$$Ab^2 - 4A$$

Factor out A:

$$A(b^2 - 4)$$

Use the difference-of-two-squares formula $C^2 - D^2 = (C + D)(C - D)$ with $C = b$ and $D = 2$:

$$A(b + 2)(b - 2)$$

Substitute $A = a^2 - 1$:

$$(a^2 - 1)(b + 2)(b - 2)$$

Use the difference-of-two-squares formula $C^2 - D^2 = (C + D)(C - D)$ with $C = a$ and $D = 1$:

$$(a + 1)(a - 1)(b + 2)(b - 2)$$

1.3.92 Factor out 3x:

$$3x(x^2 - 9)$$

Use the difference-of-two-squares formula $A^2 - B^2 = (A + B)(A - B)$ with $A = x$ and $B = 3$:

$$3x(x + 3)(x - 3)$$

1.3.94 Factor out x^2 from the first two terms:

$$x^2(x + 3) - x - 3$$

Factor out -1 from the last two terms:

$$x^2(x + 3) - (x + 3)$$

Let $A = x + 3$:

$$x^2A - A$$

Factor out A:

$$A(x^2 - 1)$$

Use the difference-of-two-squares formula $A^2 - B^2 = (A + B)(A - B)$ with $A = x$ and $B = 1$:

$$A(x + 1)(x - 1)$$

Substitute $A = x + 3$:

$$(x + 3)(x + 1)(x - 1)$$

1.3.97 Let $A = x - 1$ and $B = x + 2$:

$$AB^2 - A^2B$$

Factor out A:

$$A(B^2 - AB)$$

Factor out B:

$$AB(B - A)$$

Substitute $A = x - 1$ and $B = x + 2$:

$$(x - 1)(x + 2)((x + 2) - (x - 1))$$

Simplify:

$$(x - 1)(x + 2)(x + 3)$$

1.3.103 Let $A = x^2 + 3$:

$$A^{-1/3} - \frac{2}{3}x^2A^{-4/3}$$

Factor out $A^{-4/3}$:

$$A^{-4/3}\left(A^1 - \frac{2}{3}x^2\right)$$

Substitute $A = x^2 + 3$:

$$(x^2 + 3)^{-4/3}\left(x^2 + 3 - \frac{2}{3}x^2\right)$$

Simplify the expression in the second set of parentheses:

$$(x^2 + 3)^{-4/3}\left(\frac{1}{3}x^2 + 3\right)$$

Make it a nice little fraction:

$$\frac{\frac{1}{3}x^2 + 3}{(x^2 + 3)^{4/3}}$$

Note: Why $-4/3$? When we factor out a term, we pull out the one with the smallest exponent. This principle is evident in the positive-exponent case: factoring out x^5 or x^3 in $x^5 + x^3 + x^2$ would be absurd. As for the new exponent (1 in this case) which seems to have replaced the rabbit in the hat, the following computational principle has been used:

(Factored-out exponent)+(New exponent)=(Old exponent).

1.3.104 Let $A = 3x + 4$:

$$\frac{1}{2}x^{-1/2}A^{1/2} - \frac{3}{2}x^{1/2}A^{-1/2}$$

Factor out $\frac{1}{2}$:

$$\frac{1}{2}(x^{-1/2}A^{1/2} - 3x^{1/2}A^{-1/2})$$

Factor out $x^{-1/2}$:

$$\frac{1}{2}x^{-1/2}(A^{1/2} - 3xA^{-1/2})$$

Factor out $A^{-1/2}$:

$$\frac{1}{2}x^{-1/2}A^{-1/2}(A - 3x)$$

Substitute $A = 3x + 4$:

$$\frac{1}{2}x^{-1/2}(3x+4)^{-1/2}(3x + 4 - 3x)$$

Simplify:

$$\frac{1}{2}x^{-1/2}(3x+4)^{-1/2}(4) = 2x^{-1/2}(3x+4)^{-1/2}$$

1.3.105 (a) Expand the expression on the right side of the equal sign:

$$\frac{1}{2}[(a^2 + 2ab + b^2) - (a^2 + b^2)]$$

Collect like terms:

$$\frac{1}{2}[(a^2 - a^2) + (b^2 - b^2) + 2ab)]$$

Simplify:

$$\frac{1}{2}[2ab] = ab$$

(b) On the left side of the equal sign, let $A = a^2 + b^2$ and $B = a^2 - b^2$:

$$A^2 - B^2$$

Use the formula $A^2 - B^2 = (A + B)(A - B)$:

$$(A + B)(A - B)$$

Substitute $A = a^2 + b^2$ and $B = a^2 - b^2$:

$$[(a^2 + b^2) + (a^2 - b^2)][(a^2 + b^2) - (a^2 - b^2)]$$

Collect like terms:

$$[(a^2 + a^2) + (b^2 - b^2)][(a^2 - a^2) - (b^2 + b^2)]$$

Simplify:

$$(2a^2)(2b^2)$$

Simplify further:

$$4a^2b^2$$

(c) Expand the expression on the left side of the equal sign:

$$(a^2c^2 + 2abcd + b^2d^2) + (a^2d^2 - 2abcd + b^2c^2)$$

Collect like terms:

$$a^2c^2 + a^2d^2 + b^2d^2 + b^2c^2 + (2abcd - 2abcd)$$

Simplify:

$$a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$$

Factor out a^2:

$$a^2(c^2 + d^2) + b^2c^2 + b^2d^2$$

Factor out b^2:

$$a^2(c^2 + d^2) + b^2(c^2 + d^2)$$

Let $E = c^2 + d^2$:

$$a^2E + b^2E$$

Factor out E:

$$E(a^2 + b^2)$$

Substitute $(c^2 + d^2)$:

$$(c^2 + d^2)(a^2 + b^2)$$

Rearrange:

$$(a^2 + b^2)(c^2 + d^2)$$

(d) Let $D = 2ac$ and $E = a^2 - b^2 + c^2$:

$$D^2 - E^2$$

Use the formula $D^2 - E^2 = (D + E)(D - E)$:

$$(D + E)(D - E)$$

Substitute $D = 2ac$ and $E = a^2 - b^2 + c^2$:

$$(2ac + (a^2 - b^2 + c^2))(2ac - (a^2 - b^2 + c^2))$$

Rearrange:

$$(a^2 + 2ac + c^2 - b^2)(b^2 - a^2 + 2ac - c^2)$$

Group ‘em strategically:

$$[(a^2 + 2ac + c^2) - b^2][b^2 - (a^2 - 2ac + c^2)]$$

Use the perfect square formuals $a^2 + 2ac + c^2 = (a + c)^2$ and $a^2 - 2ac + c^2 = (a - c)^2$:

$$[(a + c)^2 - b^2][b^2 - (a - c)^2]$$

Use the difference-of-two-squares formula:

$$(a + c + b)(a + c - b)(b + a - c)(b - a + c)$$

2 Grading

Six problems are graded each week. As a justification for the choice of problems I grade, I shall provide, in this section, a somewhat detailed description of the graded problems. Each graded problem is worth 1 point: to receive the point, the problem must be correct. That is, no committing-two-errors-and-coming-back-to-the-right-track magic trick shall be permitted. The rest of the problem set is graded based on the number of problems completed; up to 4 points can be earned from this portion of the homework.

1.3.21 is the grade-booster of the week, being the easiest problem in the problem set. The problem is straightforward, and it does not require one to recall any particular formula. **1.3.15** is very similar, but is arithmetically trickier.

1.3.32 is the most computationally difficult one in the sequence of problems (**1.3.29**, **1.3.30**, **1.3.31**, and **1.3.32**) that *suggest* you to apply the difference-of-two-squares formula. The sequence rewards those who understood this objective, as it gets progressively more difficult to FOIL out the expression.

1.3.75 checks one's basic understanding of the factoring techniques. For those who did not do this problem correctly, I recommend doing **1.3.73**, **1.3.74**, **1.3.76**, **1.3.77**, **1.3.78**, and **1.3.79**. (This is a rather *strong* recommendation, as one cannot afford to subsist on poor factoring techniques in a precalculus course.)

1.3.86 is a slightly more difficult factoring problem. It is more substantial than **1.3.92**, **1.3.94**, or **1.4.97**.

1.3.104 includes the dreaded fractional exponents. Although computationally difficult, this problem involves surprisingly few *actual* factoring techniques; in fact, just one.

1.3.105d is a culmination the factoring techniques one ought to master in any given precalculus course. This problem is quite difficult, as multiple techniques are used, one after another.