Problem 1. Perform the indicated operation and simplify (3 points).

$$5(3t-4)-(t^2+2)-2t(t-3)$$

Solution. Expand the first set of parentheses:

$$15t-20-(t^2+2)-2t(t-3)$$

Expand the second set of parentheses:

$$15t-20-t^2-2-2t(t-3)$$

Expand the third set of parentheses:

$$15t-20-t^2-2-2t^2+6t$$

Collect like terms:

$$(-1-2)t^2+(15+6)t+(-20-2)$$

Simplify:

$$-3t^2+21t-22$$

□

Problem 2. Simplify the following expression by multiplying the factors (3 points).

$$(\sqrt[3]{5}-1)(\sqrt[3]{5^2}+\sqrt[3]{5}+1)$$

Solution 1. Pull out the "square" in $\sqrt[3]{5^2}$:

$$(\sqrt[3]{5}-1)(\sqrt[3]{5}^2+\sqrt[3]{5}+1)$$

Let $A=\sqrt[3]{5}-1$ and $B=1$:

$$(A-B)(A^2+AB+B^2)$$

Use the formula $A^3-B^3=(A-B)(A^2+AB+B^2)$:

$$A^3-B^3$$

Substitute $A=\sqrt[3]{5}-1$ and $B=1$:

$$\sqrt[3]{5}^3-1^3$$

Simplify:

$$5-1=4$$

□

Solution 2. Expand:

$$\sqrt[3]{5}\cdot\sqrt[3]{5^2}+\sqrt[3]{5}\cdot\sqrt[3]{5}+\sqrt[3]{5}\cdot 1-1\cdot\sqrt[3]{5^2}-1\cdot\sqrt[3]{5}-1\cdot 1$$

Simplify each term:

$$\sqrt[3]{5^3}+\sqrt[3]{5^2}+\sqrt[3]{5}-\sqrt[3]{5^2}-\sqrt[3]{5}-1$$

Collect like terms:

$$\sqrt[3]{5^3}+(\sqrt[3]{5^2}-\sqrt[3]{5^2})+(\sqrt[3]{5}-\sqrt[3]{5})-1$$

Simplify:

$$\sqrt[3]{5^3}-1=5-1=4$$

□

Problem 3. Factor the expression completely (4 points).

$$\left(1+\frac{1}{t}\right)^2-\left(1-\frac{1}{t}\right)^2$$

Solution 1. Let $A = 1 + \frac{1}{t}$ and $B = 1 - \frac{1}{t}$:

$$A^2 - B^2$$

Use the formula $A^2 - B^2 = (A + B)(A - B)$:

$$(A + B)(A - B)$$

Substitute $A = 1 + \frac{1}{t}$ and $B = 1 - \frac{1}{t}$:

$$\left[\left(1+\frac{1}{t}\right)+\left(1-\frac{1}{t}\right)\right]\left[\left(1+\frac{1}{t}\right)-\left(1-\frac{1}{t}\right)\right]$$

Expand the parentheses inside the square brackets:

$$\left[1+\frac{1}{t}+1-\frac{1}{t}\right]\left[1+\frac{1}{t}-1+\frac{1}{t}\right]$$

Collect like terms:

$$\left[(1+1)+\left(\frac{1}{t}-\frac{1}{t}\right)\right]\left[(1-1)+\left(\frac{1}{t}+\frac{1}{t}\right)\right]$$

Simplify:

$$(2)\left(\frac{2}{t}\right)=\frac{4}{t}$$

□

Solution 2. Let $A = 1$ and $B = \frac{1}{t}$:

$$(A + B)^2 - (A - B)^2$$

Use the formulas $(A + B)^2 = A^2 + 2AB + B^2$ and $(A - B)^2 = A^2 - 2AB + B^2$:

$$A^2 + 2AB + B^2 - (A^2 - 2AB + B^2)$$

Expand:

$$A^2 + 2AB + B^2 - A^2 + 2AB - B^2$$

Collect like terms:

$$(A^2 - A^2) + (2AB + 2AB) + (B^2 - B^2)$$

Simplify:

$$4AB$$

Substitute $A = 1$ and $B = \frac{1}{t}$:

$$4(1)\left(\frac{1}{t}\right)$$

Simplify:

$$\frac{4}{t}$$

□

Remark. I have qualms about *Solution 2* in **Problem 3**, despite that it requires significantly less brainpower (it is, after all, just expanding the expression, while secretly hoping that something simple will eventually appear on the paper): **Problem 3** is a factoring problem, and *Solution 2* takes the opposite approach (namely, expanding). Fortunately, going the wrong way worked in this particular problem. In general, however, it can lead you to serious problems, e.g., turning the expresison into a hopelessly complicated one.