

Problem 1. Use the method of Completing the Square to solve $2x^2 = 10x - 9$ (4 points).

Solution. Subtract $10x$ from both sides:

$$2x^2 - 10x = -9$$

Factor out 2:

$$2(x^2 - 5x) = -9$$

Add $(-5 \div 2)^2 = \frac{25}{4}$ inside the parentheses:

$$2\left(x^2 - 5x + \frac{25}{4}\right) = -9 + 2 \cdot \frac{25}{4}$$

We now have a perfect square:

$$2\left(x - \frac{5}{2}\right)^2 = -9 + 2 \cdot \frac{25}{4}$$

Simplify the right side:

$$2\left(x - \frac{5}{2}\right)^2 = \frac{7}{2}$$

Divide by 2:

$$\left(x - \frac{5}{2}\right)^2 = \frac{7}{4}$$

Take the square root:

$$x - \frac{5}{2} = \pm \frac{\sqrt{7}}{2}$$

Add $\frac{5}{2}$:

$$x = \frac{5}{2} \pm \frac{\sqrt{7}}{2}$$

Simplify:

$$x = \frac{1}{2}(5 + \sqrt{7})$$

Problem 2. Solve the following equation (6 points):

$$\frac{x}{2x+7} - \frac{x+1}{x+3} = 1.$$

Solution. Multiply by the least common denominator, $(2x+7)(x+3)$:

$$x(x+3) - (x+1)(2x+7) = (2x+7)(x+3)$$

Expand the first set of parentheses:

$$x^2 + 3x - (x+1)(2x+7) = (2x+7)(x+3)$$

Expand the second set of parentheses:

$$x^2 + 3x - (2x^2 + 9x + 7) = (2x+7)(x+3)$$

Expand the third set of parentheses:

$$x^2 + 3x - (2x^2 + 9x + 7) = 2x^2 + 13x + 21$$

Subtract $2x^2 + 13x + 21$:

$$x^2 + 3x - (2x^2 + 9x + 7) - (2x^2 + 13x + 21) = 0$$

Collect like terms;

$$(1 - 2 - 2)x^2 + (3 - 9 - 13)x + (-7 - 21) = 0$$

Simplify:

$$-3x^2 - 19x - 28 = 0$$

Multiply by -1 :

$$3x^2 + 19x + 28 = 0$$

Factor it, though it may look like (*do it!*) like magic:

$$(x + 4)(3x + 7) = 0$$

Solve the equation:

$$x = -4, -\frac{7}{3}$$