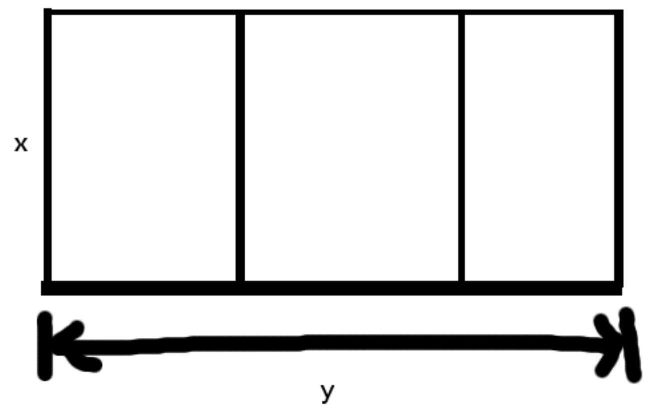
Name:______________________ **Section**:__________

Problem 1. A farmer has 1800 ft. of fencing available to build 3 adjacent pens.



(a) Express the total area of the pens as a function of a side x. (2 points.)

Solution. The total area of the pens is xy. It remains to express y in terms of x. The total length of the fencing is 1800 ft., hence $4x + 2y = 1800$. It follows that $y = -2x + 900$. Substituting, we get $A(x) = x(-2x + 900) = -2x^2 + 900x$.

(b) Find the domain of the area function you found in (a). Hint: use the fact that $y > 0$. (2 points.)

Solution. If $x \leq 0$, then we cannot make any pen. (Could you make a pen with negative-length fencing?). Therefore, $x > 0$. In addition, if $4x \geq 1800$, then we cannot close off the pens. Hence, $4x < 1800$, which, in turn, implies that $x < 450$. It follows that the domain of $A(x)$ is $(0, 450)$, or $\{x : 0 < x < 450\}$.

(c) Find the dimensions of x and y that produces the largest area. You must show your work to receive full credit. (4 points.)

Solution.

$$\begin{aligned} A(x) &= -2x^2 + 900x \\ &= -2(x^2 - 450x) \\ &= -2(x^2 - 450x + 50625 - 50625) \\ &= -2(x^2 - 450x + 50625) - 2 \cdot -50625 \\ &= -2(x - 225)^2 + 101250 \end{aligned}$$

The vertex of the parabola is $(225, 101250)$, at which $A(x)$ attains its maximum. Hence, the x-value of the maximized pens is 225 ft. The y-value is $101250 \div 225 = 450$.

(d) Find the largest total area of the pens.

Solution. We have already found the largest total area of the pens in (c). At $x = 225$, the area function $A(x)$ attains its maximum; $A(225) = 101250$.