

Problem 1 (1 points). Solve $6x^2 - x - 15 = 0$ by factoring the left-hand side.

Solution. $(3x - 5)(2x + 3) = 0$, and so $x = \frac{5}{3}, -\frac{3}{2}$. □

Problem 2 (2 points). Solve $6x^2 - x - 15 = 0$ by completing the square.

Solution. Divide through by 6:

$$x^2 - \frac{1}{6}x - \frac{5}{2} = 0.$$

Add $\frac{5}{2}$:

$$x^2 - \frac{1}{6}x = \frac{5}{2}.$$

Dividing $\frac{1}{6}$ by 2, we get $\frac{1}{12}$. Squaring, we get $\frac{1}{144}$. So, add $\frac{1}{144}$ to both sides:

$$x^2 - \frac{1}{6}x + \frac{1}{144} = \frac{5}{2} + \frac{1}{144}.$$

Substitute $\frac{5}{2} + \frac{1}{144} = \frac{361}{144}$:

$$x^2 - \frac{1}{6}x + \frac{1}{144} = \frac{361}{144}.$$

Group the left side; the constant term is $-\frac{1}{6} \div 2 = \frac{1}{12}$:

$$\left(x - \frac{1}{12}\right)^2 = \frac{361}{144}.$$

Take the square root:

$$x - \frac{1}{12} = \pm \sqrt{\frac{361}{144}}.$$

Substitute $\sqrt{361} = 19$ and $\sqrt{144} = 12$:

$$x - \frac{1}{12} = \pm \frac{19}{12}.$$

Add $\frac{1}{12}$:

$$x = \frac{1}{12} \pm \frac{19}{12}.$$

It thus follows that

$$x = \frac{5}{3}, -\frac{3}{2}$$

□

Problem 3 (1 point). State the quadratic formula for the general quadratic equation $ax^2 + bx + c = 0$.

Solution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

□

Problem 4 (1 point). Solve $6x^2 - x - 15 = 0$ using the quadratic formula.

Solution.

$$x = \frac{-1 \pm \sqrt{1^2 - (4)(6)(-15)}}{12} = \frac{-1 \pm \sqrt{361}}{12} = \frac{-1 \pm 19}{12} = \frac{5}{3}, -\frac{3}{2}.$$

□

Problem 5 (5 points). Patty Berglund is cooking a pot of stew for her son, Joey. To simplify the problem, let us assume that the stew consists of water and Miracle Sauce B. Miracle Sauce B, unlike the outdated Miracle Sauce A, is clearly the most delicious thing in the world, so Patty wants to put as much into the stew as possible. Unfortunately, if more than 30% of the stew is Miracle Sauce B, then the stew turns itself into a glob of jelly. Nobody wants to see this, of course, so Patty resolves to make stew consisting of 70% water and 30% Miracle Sauce B.

Patty puts some water and Miracle Sauce B into the pot, only to realize that the stew is 90% water and 10% Miracle Sauce B. Alas, Patty says, mistakes were made! As Joey specifically requested for 1L of stew, she cannot simply add more Miracle Sauce B: she must boil off some water, and then add more Miracle Sauce B.

- (i) (3 points) How much water should be boiled off and replaced with Miracle Sauce B? (Remember, the unit is *liter*.)

Solution. Before we change anything, 90% of 1L is water, so we have 0.9L of water. Similarly, 10% of 1L is Miracle Sauce B, so we have 0.1L of Miracle Sauce B. We want 70% of 1L to be water, and 30% of 1L to be Miracle Sauce B: this means we want 0.7L of water, and 0.3L of Miracle Sauce B. Therefore, we should boil off

$$0.9 - 0.7 = 0.2$$

liters of water.

□

- (ii) (2 points) If the boil-off rate of water is 4L per hour, for how long should the water be boiled off?

Solution. Since (hours required) = (liters) \div (liters per hour), we need to boil off the water for

$$0.2 \div 4 = 0.05$$

hours. Using (minutes) = (hour) \times 60, we conclude that the water should be boiled off for

$$0.05 \times 60 = 3$$

minutes.

□