Problem 1. Solve $x^2 - x - 20 \geq 0$.

Solution. The above inequality is equivalent to

$$(x-5)(x+4) \geq 0,$$

and so

$$x \geq 5 \text{ or } x \leq -4.$$

□

Problem 2. Solve $2x^2 + 3x - 2 \leq 0$.

Solution. The above inequality is equivalent to

$$(2x-1)(x+2) \leq 0,$$

and so

$$-2 \leq x \leq \frac{1}{2}.$$

□

Problem 3. Solve $x^2 + 5x + 6 > 0$.

Solution. The above inequality is equivalent to

$$(x+2)(x+3) > 0,$$

and so

$$x > -2 \text{ or } x < -3.$$

□

Problem 4. Solve $6x^2 - 17x + 12 < 0$.

Solution. The above inequality is equivalent to

$$(2x-3)(3x-4) < 0,$$

and so

$$\frac{4}{3} < x < \frac{3}{2}.$$

□

Problem 5. Solve $(x+2)(x-1)(x-3) \geq 0$. (Hint: Read "A Counting Magic For Inequalities" from Week 4.)

Solution. The critical points are $x = -2, 1, 3$, whence the intervals we should consider are $(-\infty, -2]$, $[-2, 1]$, $[1, 3]$, and $[3, \infty)$. By a flipping argument, $(x+2)(x-1)(x-3)$ is nonnegative on $[-2, 1]$ and $[3, \infty)$. Therefore,

$$x \in [-2, 1] \cup [3, \infty).$$

□

Problem 6. Find the equation of the line through $(-1,-2)$ and $(4,3)$.

Solution. We consider a generic line

$$y = mx + b.$$

In this case,

$$m = \frac{4-(-1)}{3-(-2)} = 1$$

and

$$b = y - mx = 3 - 4 = -1.$$

It follows that the desired line is

$$y = x - 1.$$

□

Problem 7. Find the equation of the line through $(1,-6)$ parallel to the line $x + 2y = 6$.

Solution. Let us put $x + 2y = 6$ into the slope-intercept form:

$$y = -\frac{1}{2}x + 3.$$

The desired line, which is parallel to the above, is of the form

$$y = -\frac{1}{2}x + b.$$

Plugging in $(x,y) = (1,-6)$, we see that

$$b = y + \frac{1}{2}x = -6 + \frac{1}{2} = -\frac{11}{2}.$$

It follows that the desired line is

$$y = -\frac{1}{2}x - \frac{11}{2}.$$

□

Problem 8. Find the equation of the line through $(-1,-2)$ perpendicular to the line $2x+5y+8 = 0$.

Solution. Let us put $2x + 5y + 8 = 0$ in the slope-intercept form:

$$y = -\frac{2}{5}x - \frac{8}{5}.$$

The desired line, which is perpendicular to the above, is of the form

$$y = \frac{5}{2}x + b.$$

Plugging in $(x,y) = (-1,-2)$, we see that

$$b = y - \frac{5}{2}x = -2 + \frac{5}{2} = \frac{1}{2}.$$

It follows that the desired line is

$$y = \frac{5}{2}x + \frac{1}{2}.$$

□