Problem 1 (10 points). Use the Laws of Logarithms to expand the following expression completely:

$$\ln\left(\sqrt{\frac{e^{x^2+4}}{(x^2+1)(x^3-7)^2}}\right).$$

Proof.

$$
\begin{aligned}
\ln\left(\sqrt{\frac{e^{x^2+4}}{(x^2+1)(x^3-7)^2}}\right) &= \frac{1}{2}\ln\frac{e^{x^2+4}}{(x^2+1)(x^3-7)^2} \\
&= \frac{1}{2}\left[\ln(e^{x^2+4})(x^2+1)^{-1}(x^3-7)^{-2}\right] \\
&= \frac{1}{2}\left[\ln e^{x^2+4} + \ln(x^2+1)^{-1} + \ln(x^3-7)^{-2}\right] \\
&= \frac{1}{2}\left[(x^2+4)\ln e - \ln(x^2+1) - 2\ln(x^3-7)\right] \\
&= \frac{1}{2}\left[x^2+4 - \ln(x^2+1) - 2\ln(x^3-7)\right].
\end{aligned}
$$

$\square$