NB. No calculators allowed.

Problem 1 (4 points)**.** A colony of E. Coli is growing in a petri dish, doubling in quantity every minute. If the initial population of the colony is 1000, what is the equation that models the growth of the population? Determine the time t at which the population count reaches 10000.

Solution. The equation that models the growth of the population is

$$f(t) = 1000 \cdot 2^t.$$

To answer the second part of the problem, we must solve

$$10000 = 1000 \cdot 2^t$$

for t. Dividing both sides by 1000, we get

$$10 = 2^t,$$

whence taking $\log_2$ on both sides yields

$$\log_2 10 = t.$$

□

Problem 2 (3 points)**.** Compare the sizes of $\log_3 11$ and $\log_5 24$, and justify your answer. *Hint*: "Compare the sizes" means one of the (mutually exclusive) following should be proved:

$$\log_3 11 < \log_5 24 \quad \text{or} \quad \log_3 11 = \log_5 24 \quad \text{or} \quad \log_3 11 > \log_5 24.$$

Solution. Noting that $11 > 9 = 3^2$ and $24 < 25 = 5^2$, we see that

$$\log_5 24 < \log_5 25 = \log_5 5^2 = 2 = \log_3 3^2 = \log_3 9 < \log_3 11.$$

□

Problem 3 (3 points)**.** Find the x- and y-intercepts of

$$y = 2 - \log_7(x + 3).$$

Solution. To find the x-intercept, we set $y = 0$ to obtain

$$0 = 2 - \log_7(x + 3)$$

and solve for x. A simple rearrangement yields

$$2 = \log_7(x + 3)$$

whence by the definition of logarithm

$$7^2 = x + 3.$$

Therefore, we have $49 = x + 3$, or

$$x = 49 - 3 = 46,$$

whence the x-intercept is $(46, 0)$. To find the y-intercept, we set $x = 0$, which yields

$$y = 2 - \log_7(0 + 3) = 2 - \log_7 3,$$

whence the y-intercept is $(0, 2 - \log_7 3)$.

□