Problem 1 (2 points)**.** A colony of S. typhimurium is growing in a petri dish, quadrupling in quantity every minute. If the initial population of the colony is 500, what is the equation that models the growth of the population? Determine the time t at which the population count reaches 7800. (*Hint*: Do not use the exponential growth model.)

Solution. The equation that models the growth of the population is

$$f(t) = 500 \cdot 4^t.$$

To answer the second part of the problem, we must solve

$$7800 = 500 \cdot 4^t$$

for t. Dividing both sides by 500, we get

$$\frac{78}{5} = 4^t,$$

whence taking $\log_4$ on both sides yields

$$\log_4 \frac{78}{5} = t.$$

□

Problem 2 (2 points)**.** A colony of S. pneumoniae is growing in a petri dish. If the initial population is 3000 and the population after 50 minutes is 15000, what is the equation that models the growth of the population? Determine the time t at which the population count reaches 30000. (*Hint*: Use the exponential growth model.)

Solution. The equation that models the growth of the population is

$$f(t) = 3000e^{rt}$$

for some r. Since $f(50) = 15000$, we have

$$15000 = 3000e^{50r}.$$

Dividing both sides by 3000, we get

$$5 = e^{50r},$$

whence taking ln on both sides yields

$$\ln 5 = 50r,$$

or

$$r = \frac{\ln 5}{50}.$$

To answer the second part of the problem, we must solve

$$30000 = 3000e^{(\ln 5/50)t}.$$

Dividing both sides by 3000, we get

$$10 = e^{(\ln 5/50)t},$$

whence taking ln on both sides yields

$$\ln 10 = \frac{\ln 5}{50} t,$$

or

$$t = \frac{50 \ln 10}{\ln 5} = 50 \log_5 10.$$

□

Problem 3 (1 point)**.** Compare the sizes of $\log_5 26$ and $\log_7 48$.

Solution. Noting that $26 > 25 = 5^2$ and $48 < 49 = 7^2$, we see that

$$\log_7 48 < \log_7 49 = \log_7 7^2 = 2 = \log_5 5^2 = \log_5 25 < \log_5 26.$$

□

Problem 4 (1 point)**.** Compare the sizes of $e^{1.9}$ and $2^{2.59}$, using the approximation $\ln 2 \approx 0.69$.

Solution. Noting that $\ln e^{1.9} = 1.9$ and $\ln 2^{2.59} = 2.59 \ln 2 \approx 1.79$, we see that

$$e^{1.9} > 2^{2.59}.$$

□

Problem 5 (2 points)**.** Find the x- and y-intercepts of

$$y = 2 - \log_5(x + 7).$$

Solution. To find the x-intercept, we set $y = 0$ to obtain

$$0 = 2 - \log_5(x + 7)$$

and solve for x. A simple rearrangement yields

$$2 = \log_5(x + 7)$$

whence by the definition of logarithm

$$5^2 = x + 7.$$

Therefore, we have $49 = x + 3$, or

$$x = 25 - 7 = 18,$$

whence the x-intercept is $(18, 0)$. To find the y-intercept, we set $x = 0$, which yields

$$y = 2 - \log_5(0 + 7) = 2 - \log_5 7,$$

whence the y-intercept is $(0, 2 - \log_5 7)$.

□