Section 1.5, Problem 49

This is a typical completing-the-squares problem. If you feel insecure about this topic, you should consider studying the **Completing the Squares: Developed as a Series of Exercises** worksheet, posted on the website.

Let us restate the original equation first:

$$2x^2 + 8x + 1 = 0$$

Divide by 2:

$$x^2 + 4x + \frac{1}{2} = 0$$

Add the divide-by-two-and-square-it term:

$$x^2 + 4x + \frac{1}{2} + 4 = 4$$

Rearrange and group the relevant terms:

$$(x^2 + 4x + 4) + \frac{1}{2} = 4$$

Factor:

$$(x + 2)^2 + \frac{1}{2} = 4$$

Subtract $\frac{1}{2}$ from both sides

$$(x + 2)^2 = \frac{7}{2}$$

Take the square root:

$$x + 2 = \sqrt{\frac{7}{2}}$$

Rationalize the denominator:

$$x + 2 = \frac{\sqrt{14}}{2}$$

Subtract 2 from both sides:

$$x = -2 + \frac{\sqrt{14}}{2}$$

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There are two important points to be considered. First, we note that

$$\frac{A}{B} = 0 \text{ if and only if } A = 0.$$

Thus, when you have a fractional equation, it suffices to combine the terms into **one** fractional expression, and then solve the equation **just for the numerator**. Two, any solution that makes the denominator zero is **unacceptable**. Such solutions must be excluded, and you must explain why they ought to be excluded.

Here is the problem:

$$\frac{x+5}{x-2} = \frac{5}{x+2} + \frac{28}{x^2-4}$$

We deal with the right-hand side first. First, we find a common denominator, noting that $x^2 - 4 = (x-2)(x+2)$:

$$\frac{x+5}{x-2} = \frac{5(x-2)}{(x+2)(x-2)} + \frac{28}{(x+2)(x-2)}$$

Expand:

$$\frac{x+5}{x-2} = \frac{5x-10}{(x+2)(x-2)} + \frac{28}{(x+2)(x-2)}$$

Add:

$$\frac{x+5}{x-2} = \frac{5x-10+28}{(x+2)(x-2)}$$

And simplify:

$$\frac{x+5}{x-2} = \frac{5x+18}{(x+2)(x-2)}$$

Now, subtract $\frac{x+5}{x-2}$ from both sides—

$$0 = \frac{5x+18}{(x+2)(x-2)} - \frac{x+5}{x-2}$$

We'll need to find a common denominator again:

$$0 = \frac{5x+18}{(x+2)(x-2)} - \frac{(x+5)(x+2)}{(x+2)(x-2)}$$

Again, expand:

$$0 = \frac{5x+18}{(x+2)(x-2)} - \frac{x^2+7x+10}{(x+2)(x-2)}$$

Add:

$$0 = \frac{5x+18-x^2-7x-10}{(x+2)(x-2)}$$

And simplify:

$$0 = \frac{-x^2-2x+8}{(x+2)(x-2)}$$

We have successfully reduced the equation to the form

$$\frac{A}{B} = 0,$$

whence we need only to solve

$$A = 0.$$

We proceed to do so.

$$-x^2 - 2x + 8 = 0$$

Multiply -1 to the both sides:

$$x^2 + 2x - 8 = 0$$

And factor it:

$$(x-2)(x+4) = 0.$$

So, we have $x = 2$ and $x = -4$. We are not done yet, however. We need to check whether our solutions make the denominator zero. Indeed, $x = 2$ makes the denominator zero, hence it cannot be a solution. We conclude that

$$x = -4.$$

See page 55, problems 75-78 for relevent exercises.

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The notion to discuss here is that of **extraneous solutions**. The gist is this: when you *transform* one equation to another, sometimes you obtain solutions that do not satisfy the original equation. An example is in order—suppose we have an equation:

$$x = 2$$

Well, the solution is clearly $x = 2$. Now, what if we square it?

$$x^2 = 4$$

Now, we have $x = 2$ and $x = -2$. We just got another solution! If our purpose is to solve the original equation, we have to throw away $x = -2$.

The moral is, of course, that you need to check to make sure the solutions acutally work. Let's get to work, then:

$$\sqrt{5-x} + 1 = x - 2$$

Our first job is to isolate the square-root term; this we do now:

$$\sqrt{5-x} = x - 2 - 1$$

Simplify the right-hand side:

$$\sqrt{5-x} = x - 3$$

Now to the dangerous step—square both sides:

$$5 - x = (x-3)^2$$

Expand the right-hand side:

$$5 - x = x^2 - 6x + 9$$

Subtract $(5-x)$ from both sides:

$$0 = x^2 - 6x + 9 - 5 + x$$

Simplify the right-hand side:

$$0 = x^2 - 5x + 4$$

Factor it:

$$0 = (x-1)(x-4)$$

So, we get $x = 1$ and $x = 4$. We must, of course, check for possible extraneous solutions. We try $x = 1$ first:

$$\sqrt{5-1} + 1 = 2 + 1 = 3$$

whereas

$$1 - 2 = -1,$$

This is not right. $x = 1$ is not a solution. Let us check $x = 4$:

$$\sqrt{5-4} + 1 = 1 + 1 = 2$$

and

$$4 - 2 = 2,$$

Certainly, $x = 4$ is a solution. Therefore, $x = 4$ is the only solution.

See page 56, problems 81, 83, and 84 for related exercises. Problem 84 is a good one.