Name:_____ **Section**:_____

Problem 1. This problem deals with the rates of change of a family of functions.

Part 1 (1). Let $f(t) = t^2$. Calculate the average rate of change of the function f between $t = x$ and $t = x + h$. (Make sure to simplify the fraction completely by cancelling out the extra h)

Part 2 (1). Plug in zero for h in the answer you produced in Part 1. What do you get?

Part 3 (1). The answer for Part 2 should be in terms of x; that is, it should be a function of x. This function, often labelled $f'(x)$, is called the *instantaneous rate of change* of the function f. Calculate the instantaneous rate of change of f at $x = 3$.

Part 4 (2). Now, let c be a constant, and let $g(t) = t^2 + c$ (Note: We have, in effect, shifted f vertically by c). Compute the instantaneous rate of change of the function g by repeating the processes outlined in Parts 1 and 2:

- Calculate the average rate of change of the function $g(t)$ between $t = x$ and $t = x + h$.
- Plug in zero for h.

Part 5 (1). Compare the instantaneous rate of change of the function g with the instantaneous rate of change of the function f. What can you conclude about these two functions?

Part 6 (2). Let $c = 5$, so that we have $g(x) = x^2 + 5$ (We are changing t to x, so as to revert back to the good-old xy-cooordinate). What is the vertex of the parabola?

Part 7 (2). At which point x does the function g attain its minimum?

Part 8 (1). Calculate the instantaneous rate of change of the function g at the point of minimum.

Extra credit (2) What can you conclude about the relationship between the instantaneous rate of change of a function and its points of minima?