

Problem 1. State the factoring formula for each of the following expressions (2 points).

$$A^2 - B^2 = (A + B)(A - B)$$

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

Problem 2. Factor the following expressions (2 points).

$$4x^2 - y^2 = (2x + y)(2x - y)$$

$$8x^3 - y^3 = (2x - y)(4x^2 + 2xy + y^2)$$

Problem 3. Simplify the following expression completely (1 point).

$$\frac{4x^2 - y^2}{8x^3 - y^3} = \frac{(2x + y)(2x - y)}{(2x - y)(4x^2 + 2xy + y^2)} = \frac{2x + y}{4x^2 + 2xy + y^2}$$

Problem 4. Simplify the following expression completely, and rationalize the denominator (5 points).

$$\begin{aligned}
& \frac{\frac{4x^2 - y^2}{8x^3 - y^3} - \frac{yz}{4x^2z + 2xyz + y^2z}}{8\sqrt[3]{x^5}} \\
& \frac{\frac{4x^2 - y^2}{8x^3 - y^3} - \frac{yz}{4x^2z + 2xyz + y^2z}}{8\sqrt[3]{x^5}} \\
& \frac{\frac{4x^2 - y^2}{8x^3 - y^3} - \frac{yz}{4x^2z + 2xyz + y^2z}}{8\sqrt[3]{x^5}} \\
& = \left(\frac{4x^2 - y^2}{8x^3 - y^3} - \frac{yz}{4x^2z + 2xyz + y^2z} \right) \div \left(\frac{8\sqrt[3]{x^5}}{8x^2 + 4xy + 2y^2} \right) \\
& = \left(\frac{(2x + y)(2x - y)}{(2x - y)(4x^2 + 2xy + y^2)} - \frac{z \cdot y}{z(4x^2 + 2xy + y^2)} \right) \div \left(\frac{8\sqrt[3]{x^5}}{8x^2 + 4xy + 2y^2} \right) \\
& = \left(\frac{2x + y}{4x^2 + 2xy + y^2} - \frac{y}{4x^2 + 2xy + y^2} \right) \div \left(\frac{8\sqrt[3]{x^5}}{8x^2 + 4xy + 2y^2} \right) \\
& = \frac{2x + y - y}{4x^2 + 2xy + y^2} \div \left(\frac{8\sqrt[3]{x^5}}{8x^2 + 4xy + 2y^2} \right) \\
& = \frac{2x}{4x^2 + 2xy + y^2} \div \left(\frac{8\sqrt[3]{x^5}}{8x^2 + 4xy + 2y^2} \right) \\
& = \frac{2x}{4x^2 + 2xy + y^2} \div \left(\frac{2 \cdot 4\sqrt[3]{x^5}}{2(4x^2 + 2xy + y^2)} \right) \\
& = \frac{2x}{4x^2 + 2xy + y^2} \div \frac{4\sqrt[3]{x^5}}{4x^2 + 2xy + y^2} \\
& = \frac{2x}{4x^2 + 2xy + y^2} \times \frac{4x^2 + 2xy + y^2}{4\sqrt[3]{x^5}} \\
& = \frac{x}{2\sqrt[3]{x^5}} \\
& = \frac{x}{2\sqrt[3]{x^5}} \times \frac{\sqrt[3]{x}}{\sqrt[3]{x}} \\
& = \frac{x\sqrt[3]{x}}{2x^2} \\
& = \frac{\sqrt[3]{x}}{2x}
\end{aligned}$$