

Problem 1. Antoine crash-landed in the middle of Sahara Desert, without a drop of water in his possession. Fortunately, he found a small well, surrounded by a number of soap trees. The well was, however, too deep, and the water too shallow. Exasperated, he looked around and found five hundred pieces of granite lying around the well. If he threw them into the well, perhaps the water will rise enough for him to drink it. If five hundred granite pieces aren't enough, however, then he must choose not to fiddle with the granite pieces, in order to keep him as hydrated as possible.

Now, the well is 15 meters deep with a circular bottom of radius 0.5 meters, and the current level of the water is 5 meters. The volume of each granite piece is 0.02 meters cubed, and the water level has to be above 14.5 meters. Given the information, should Antoine throw the stones into the well? Justify your answer. (10 points).

Hint: At any given moment, (volume) = (base area) \times (height). Hence, an increase in volume causes an increase in height (the water level).

Solution. Before the labor commences, the volume of the well water is

$$(.5 \times .5 \times \pi) \times 5 \approx 3.93 \text{ meters cubed.}$$

The volume of each granite piece is 0.02 meters cubed, and there are five hundreds of them; hence, the maximum possible total volume is

$$3.93 + 0.02 \times 500 \approx 13.93 \text{ meters cubed.}$$

At this moment, the water level is

$$13.93 \div (.5 \times .5 \times \pi) \approx 17.74 \text{ meters.}$$

Clearly, $17.74 > 14.5$. It follows that Antoine should throw the granite pieces into the well.

Solution 2. Model the equation given in the hint by

$$V = A \times H.$$

We want the water level to be at least 14.5, and the initial water level is 5, so we set $H = 14.5 - 5 = 9.5$. The base area A is given by

$$A = .5 \times .5 \times \pi \approx 0.79$$

Now, we can compute V :

$$V = A \times H = 0.79 \times 9.5 \approx 7.51.$$

Each granite piece has volume 0.02, hence we need

$$7.51 \div 0.02 \approx 376$$

pieces of granite. Hence, Antoine should throw the granite pieces into the well.