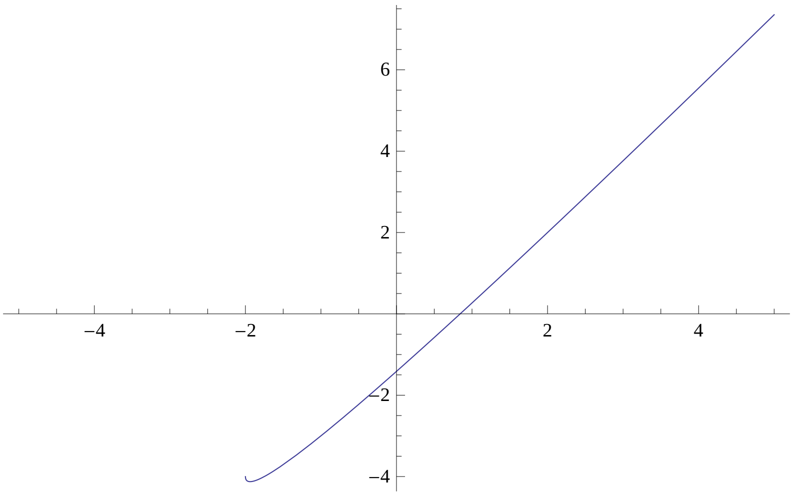
Problem 1. Solve the following equation graphically in the interval $[-2, 4]$, correct to two decimals.

$$2x - \sqrt{x+2} = 0.$$

Solution. Observe the following graph:



Clearly, there is only one value of x that satisfies $2x - \sqrt{x+2} = 0$ on the interval $[-2, 4]$. Use 2:zero in CALC F4 to deduce that

$$x \approx 0.84.$$

Problem 2. Find an equation of the line that satisfies the given conditions:

x-intercept: -7; y-intercept: -2

Solution. We start with the slope-intercept form

$$y = mx + b.$$

The y-intercept is -2, hence $b = -2$. We substitute $b = -2$ and $(x, y) = (-7, 0)$ to find m:

$$\begin{aligned} 0 &= m \cdot -7 - 2 \\ 2 &= -7m \\ -\frac{2}{7} &= m \end{aligned}$$

We conclude that the desired equation of the line is

$$y = -\frac{2}{7}x - 2.$$

Problem 3. The manager of a cheesecake factory finds that it costs \$230 to bake 100 cheesecakes in one day and \$430 to bake 400 cheesecakes in one day. Assuming that the relationship between cost and the number of cheesecakes baked is linear, find an equation that expresses this relationship. Then graph the equation.

Solution. We set the x-axis to represent the number of cheesecakese, and y-axis the cost. Then the desired line passes through $(100, 230)$ and $(400, 430)$. We may thus compute the slope:

$$m = \frac{430 - 230}{400 - 100} = \frac{200}{300} = \frac{2}{3}.$$

Hence, we have

$$y = \frac{2}{3}x + b.$$

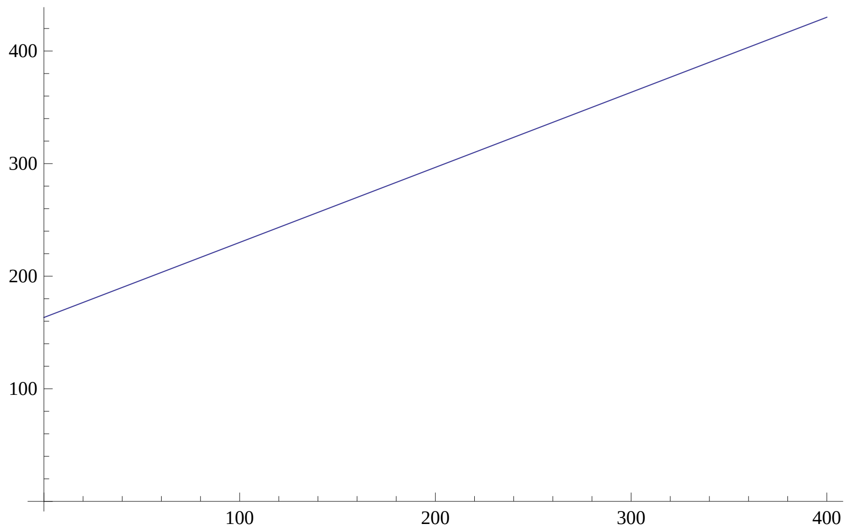
Plug in $(x, y) = (100, 230)$ to compute b:

$$\begin{aligned} 230 &= \frac{2}{3} \cdot 100 + b \\ 230 &= \frac{200}{3} + b \\ 230 - \frac{200}{3} &= b \\ \frac{490}{3} &= b \end{aligned}$$

The desired equation is, therefore:

$$y = \frac{2}{3}x + \frac{490}{3}.$$

We may graph it as thus:



Problem 4. Let $f(x) = x^3 - 4x^2$. Compute the following:

$$f(0), f(3), f(-3), f(\sqrt{3}), f(x+2), f(-x), f(x^2), f\left(\frac{x}{3}\right), 2f(x)$$

Solution.

- $f(0) = 0^3 - 4 \cdot 0^2 = 0$
- $f(3) = 3^3 - 4 \cdot 3^2 = -9$
- $f(-3) = (-3)^3 - 4 \cdot (-3)^2 = -63$
- $f(\sqrt{3}) = (\sqrt{3})^3 - 4 \cdot (\sqrt{3})^2 = -12 + 3\sqrt{3}$
- $f(x+2) = (x+2)^3 - 4(x+2)^2 = x^3 + 6x^2 + 12x + 8 - 4(x^2 + 4x^2 + 4) = x^3 + 2x^2 - 4x - 8$
- $f(-x) = (-x)^3 - 4(-x)^2 = -x^3 - 4x^2$
- $f(x^2) = (x^2)^3 - 4(x^2)^2 = x^6 - 4x^4$
- $f\left(\frac{x}{3}\right) = \left(\frac{x}{3}\right)^3 - 4\left(\frac{x}{3}\right)^2 = \frac{x^3}{27} - \frac{4x^2}{9}$
- $2f(x) = 2(x^3 - 4x^2) = 2x^3 - 8x^2$

Problem 5. Find the domain of the following function

$$g(x) = \sqrt{9 - x^2} + \frac{x}{\sqrt{2 - x}}$$

Solution. We analyze the domain of each part of the function.
Part 1. $\sqrt{9 - x^2}$
We cannot have a square root of a negative number, hence $9 - x^2 \geq 0$. Factoring, we get $(3+x)(3-x) \geq 0$. We make a table, using cut points $x = 3$ and $x = -3$:

	$x < -3$	$-3 < x < 3$	$3 < x$
$3 + x$	-	+	+
$3 - x$	+	+	-
$(3+x)(3-x)$	-	+	-

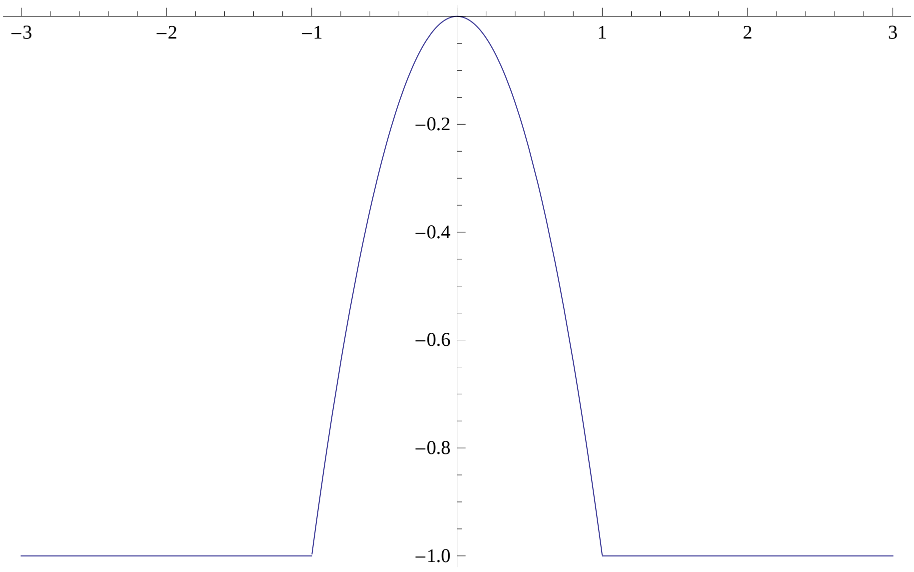
It follows from the table that $(3+x)(3-x) \geq 0$ is equivalent to $-3 \leq x \leq 3$.
Part 2. x.
There is no restriction for this part.
Part 3. $\frac{1}{\sqrt{2-x}}$
We cannot have a square root of a negative number, hence $2 - x \geq 0$. Furthermore, we cannot have 0 in the denominator, hence $2 - x > 0$. Solving the inequality yields $2 > x$.

Now, we need to find the **intersection** of $-3 \leq x \leq 3$ and $2 > x$. $[-3, 3] \cap (-\infty, 2) = [-3, 2)$, as you can check. It follows that the domain of g is $[-3, 2)$.

Problem 6. Sketch the following piecewise-defined function

$$f(x) = \begin{cases} -x^2 & \text{if } |x| \le 1 \\ -1 & \text{if } |x| > 1 \end{cases}$$

Solution. The following:



Problem 7. Determine the average rate of change of the function between the given values of the variable.

$$f(x) = 3 - 2x^2; \qquad x = -1, x = 4$$

Solution. Recall that the definition of the average of change of the function f from $x = a$ and $x = b$ is

$$\frac{f(b) - f(a)}{b - a}$$

Hence,

$$\begin{aligned} \frac{f(4) - f(-1)}{4 - (-1)} &= \frac{[3 - 2(4)^2] - [3 - 2(-1)^2]}{5} \\ &= \frac{-30}{5} \\ &= -6 \end{aligned}$$

Problem 8. Explain how the graph of g is obtained from the graph of f.

$$f(x) = x^2, \qquad g(x) = 3(x - 2)^2 + 3$$

Solution. We adhere to the following procedure:

1. Shift to the right by 2: $f(x) = x^2 \Rightarrow f(x-2) = (x-2)^2$
2. Scale it threefold: $f(x-2) = (x-2)^2 \Rightarrow 3f(x-2) = 3(x-2)^2$
3. Shift up by 3: $3f(x-2) = 3(x-2)^2 \Rightarrow 3f(x-2)+3 = 3(x-2)^2 + 3$

Problem 9. Find the vertex, x-intercept, and y-intercept of the following function

$$f(x) = 3x^2 + 6x - 4$$

Solution. The y-intercept is -4.
The x-value of the vertex is

$$-\frac{b}{2a} = -\frac{6}{3 \cdot 2} = -1.$$

Then, the y-value of the vertex is

$$f(-1) = 3(-1)^2 + 6(-1) - 4 = -7.$$

Thus, the vertext $(-1, -7)$.
Use the graphing calculator to conclude that the x-intercepts are $(-2.52, 0)$ and $(0.52, 0)$.

Problem 10. Find the local maximum and minimum values of the function and the value of x at which each occurs.

$$f(x) = x\sqrt{4-x}$$

Solution. Use the graphing calculator to conclude that f has a maximum at $x = 8/3 \approx 2.67$. The maximum value of f is

$$f(8/3) \approx 3.08.$$