**Problem 1.** Let $f(x) = \frac{2}{x}$ and $g(x) = \sqrt{x^2 - 9}$.

Part 1 (2) Find $f + g$ and its domain.

Solution

$$(f+g)(x) = \frac{2}{x} + \sqrt{x^2 - 9}$$

To compute its domain, we first note that $x \neq 0$; otherwise, $f(x)$ has a zero-denominator. Another restriction is $x^2 - 9 \geq 0$, which is equivalent to $x^2 \geq 9$, or

$$x \geq 3 \text{ or } x \leq -3.$$

Hence, the domain is $(-\infty, -3] \cup [3, \infty)$.

Part 2 (3) Find $fg$ and its domain.

Solution

$$(fg)(x) = \frac{2\sqrt{x^2 - 9}}{x}$$

The domain is identical as in Part 1.

Part 3 (5) Find $f \circ g$ and its domain.

Solution

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= \frac{2}{\sqrt{x^2 - 9}}\end{aligned}$$

We have two restrictions: (1) $x^2 - 9 \geq 0$, and (2) $x^2 - 9 \neq 0$. Combining (1) and (2), we may conclude that the domain satisfies $x^2 - 9 > 0$, or

$$x > 3 \text{ or } x < -3.$$

Hence, the domain is $(-\infty, -3) \cup (3, \infty)$.