Challenge Problem Set 2, Math 292 Spring 2012

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1 Introduction

This challenge problem set is about driven oscillatory systems.

Let $K := \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$. The object is to find and study the solution of

$$\mathbf{x}''(t) = -K\mathbf{x}(t) + \mathbf{f}(t) \quad \text{with} \quad \mathbf{x}(0) = (1,1) \quad \text{and} \quad \mathbf{x}'(0) = (0,2)\ , \tag{1.1}$$

where

$$\mathbf{f}(t) = \sum_{j=1}^{4} \mathbf{f}_j(t)$$

and

$$\begin{aligned}
\mathbf{f}_1(t) &= (1,3)\cos(\omega_1 t) \\
\mathbf{f}_2(t) &= (1,-1)\sin(\omega_2 t) \\
\mathbf{f}_3(t) &= (3,-1)\sin(\omega_3 t) \\
\mathbf{f}_4(t) &= (1,0)\cos(\omega_4 t)
\end{aligned}$$

This problem will be solved by teamwork and a divide-and-conquer strategy. One of the very nice features of such systesm is that they can be dealt with this way, as a consequence of linearity.

1. Task for team 1: Solve the homogenous system

$$\mathbf{x}''(t) = -K\mathbf{x}(t) \quad \text{with} \quad \mathbf{x}(0) = (1,1) \quad \text{and} \quad \mathbf{x}'(0) = (0,2)\ .$$

2. Task for team 2: Solve the inhomogenous system

$$\mathbf{x}''(t) = -K\mathbf{x}(t) + \mathbf{f}_1(t) \quad \text{with} \quad \mathbf{x}(0) = (0,0) \quad \text{and} \quad \mathbf{x}'(0) = (0,0)\ .$$

3. Task for team 3: Solve the inhomogenous system

$$\mathbf{x}''(t) = -K\mathbf{x}(t) + \mathbf{f}_2(t) \quad \text{with} \quad \mathbf{x}(0) = (0,0) \quad \text{and} \quad \mathbf{x}'(0) = (0,0)\ .$$

4. Task for team 4: Solve the inhomogenous system

$$\mathbf{x}''(t) = -K\mathbf{x}(t) + \mathbf{f}_3(t) \quad \text{with} \quad \mathbf{x}(0) = (0,0) \quad \text{and} \quad \mathbf{x}'(0) = (0,0)\ .$$

5. Task for team 5: Solve the inhomogenous system

$$\mathbf{x}''(t) = -K\mathbf{x}(t) + \mathbf{f}_4(t) \quad \text{with} \quad \mathbf{x}(0) = (0,0) \quad \text{and} \quad \mathbf{x}'(0) = (0,0)\ .$$

For the rest, take the solutions generated by the different teams, and combine them to answer the following questions. (Everybody should have the answers to these in thier own write-ups.)

6: Write down the general solution of (1.1). For which values of $\omega_1, \omega_2, \omega_3$ and ω_4 does the system exhibit resonance?

7: Let $\omega_1 = 1$, $\omega_2 = 1.65$, $\omega_3 = 2.65$ and $\omega_4 = 3.75$. Then one of the five functions that add up to give the solution is much larger than the others, and the solution is well-represented by keeping just this main term. What is the main term that gives this approximate solution?