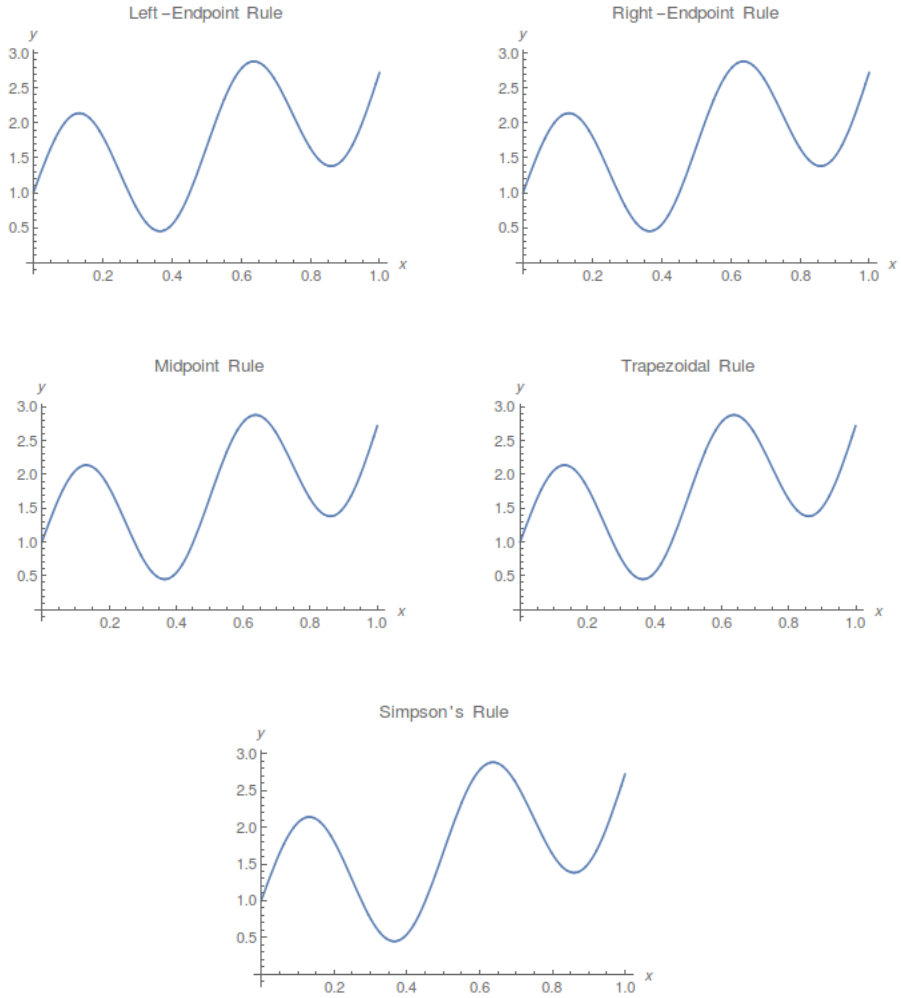
CALCULUS II, SUMMER 2015 - EXAM 1

130 points total = 120 points + 10 extra credit points

Name: _____ Score: _____/ 120

N.B. *Do not include scratch-work, but do write neatly and legibly. Everything written on these pages will be graded, so long as it is legible.*

Problem 1 (20 points). Draw graphical representations of the following approximate integration methods performed on $[0, 1]$, split into four subintervals; be sure to label the endpoints of each subinterval correctly.



Date: July 16, 2015.

Problem 2 (20 points). Compute

$$\int_{e^e}^{t} \frac{(\ln\ln\ln x)^2 + 4}{x \ln x \ln\ln x}\, dx$$

for an arbitrary choice of $t \geq e^e$. Does

$$\int_{e^e}^{\infty} \frac{(\ln\ln\ln x)^2 + 4}{x \ln x \ln\ln x}\, dx$$

converge?

Problem 3 (20 points). Compute the area of the region enclosed by the following curves:

$$y = \frac{1}{x^2+1}, \qquad y = x^2 + 1, \qquad y = -x + 3, \qquad \text{and} \qquad x = \sqrt{3}.$$

Hint: the only real root of the equation

$$\frac{1}{x^2+1} = -x + 3$$

is approximately 2.8933.

Problem 4 (20 points). Find the volume of the solid obtained by rotating the region bounded by

$$y = \frac{1}{x^2 - 3x + 2}, \qquad y = 0, \qquad x = 3, \qquad \text{and} \qquad x = 4.$$

with respect to the x-axis.

Problem 5 (20 points). Compute the arc length of the curve

$$y = \ln x$$

from $(1, 0)$ to $(e, 1)$.

Hint: It helps to make a preliminary substitution by setting $u = \frac{1}{x}$.

Problem 6 (20 points). Derive the formula for the surface area of a solid torus with the outer radius R and the inner radius r.

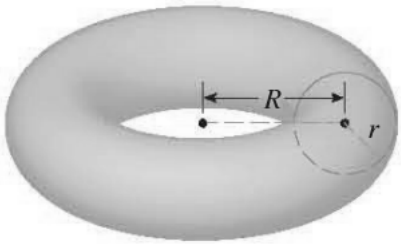


Image credit: James Stewart, *Essential Calculus: Early Transcendentals* (2e)

Bonus Problem (10 points). Integrate

$$\int \frac{\sin t}{1 + 2\sin t + \cos t}\, dt.$$