CALCULUS II, SUMMER 2015 - ODE WORKSHEET 4

Problem 1. Recall that two functions f and g are *linearly dependent* if there exists a constant c such that

$$f(x) = cg(x)$$

for all x. If there is no such c, then f and g are said to be *linearly independent*.

Determine whether the following functions are linearly dependent or linearly independent.

(1) 1 and x
(2) $\sin x$ and $\cos x$
(3) e^x and e^{-x}
(4) $\sin x$ and $\sin x \cos x$
(5) $\sin 2x$ and $\sin x \cos x$
(6) $\ln 2x$ and $4 \ln x + \ln 16$

Problem 2. A *linear combination* of two functions f and g is a function of the form

$$c_1 f(x) + c_2 g(x),$$

where c_1 and c_2 are constants. Show that f and g are linearly independent if and only if

$$c_1 f(x) + c_2 g(x) = 0$$

for all x implies that $c_1 = c_2 = 0$.

Problem 3. Find all solutions of the following differential equations

(1) $y'' - 4y = 0;$
(2) $y'' + 4y = 0;$
(3) $y'' - 4y' = 0;$
(4) $y'' + y' = 0;$
(5) $y'' - 2y' + 3y = 0;$
(6) $y'' + 2y' - 3y = 0;$
(7) $y'' - 2y' + 2y = 0;$
(8) $y'' - 2y' + 5y = 0;$
(9) $y'' + 2y' + y = 0;$
(10) $y'' - 2y' + y = 0.$

Problem 4. Find the particular solution satisfying the given initial conditions.

(1) $2y'' + 3y' = 0$ with $y = 1$ and $y' = 1$ at $x = 0$;
(2) $y'' + 25y = 0$ with $y = -1$ and $y' = 0$ at $x = 3$;
(3) $y'' - 4y' - y = 0$ with $y = 2$ and $y' = -1$ at $x = 1$;
(4) $y'' + 4y' + 5y = 0$ with $y = 2$ and $y' = y''$ at $x = 0$.

`Suggested reading`: Apostol, §§8.8 - 8.14, §§8.18

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