CALCULUS II, SUMMER 2015 - WEEKEND PROBLEM SET 2

60 points total = 50 points + 10 extra credit points

Name: _____ Score: _____/ 50

Use this worksheet as the cover sheet for your write-up: write your name on this page, and staple this sheet to the front of your homework packet.

Please indicate clearly which problems you have worked on. You will receive no credit for submitting solutions that the grader cannot read and understand—be sure to write legibly!

1. The "Paradox" of Gabriel's Horn (10 points)

Let $f(x) = \frac{1}{x}$.

Problem 1.1 (2 points)**.** Sketch the solid obtained by rotating f with respect to the x-axis on $[1, t]$. The infinitely-long horn-shaped object obtained by sending $t \to \infty$ is called *Gabriel's horn*.

Problem 1.2 (4 points)**.** What is the volume of Gabriel's horn?

Problem 1.3 (4 points)**.** What is the surface area of Gabriel's horn?

2. Exam 1: A Look Back (50 points)

Re-do the following exam problems.

Problem 2.1 (10 points)**.** Compute

$$\int_{e^e}^{t} \frac{(\ln\ln\ln x)^2 + 4}{x \ln x \ln\ln x}\, dx$$

for an arbitrary choice of $t \geq e^e$. Does

$$\int_{e^e}^{\infty} \frac{(\ln\ln\ln x)^2 + 4}{x \ln x \ln\ln x}\, dx$$

converge?

Hint: One well-chosen substitution suffices.

Problem 2.2 (10 points)**.** Compute the area of the region enclosed by the following curves:

$$y = \frac{1}{x^2+1}, \qquad y = x^2 + 1, \qquad y = -x + 3, \qquad \text{and} \qquad x = \sqrt{3}.$$

Hint: Graph all four curves, using the fact that the only real root of the equation

$$\frac{1}{x^2+1} = -x + 3$$

is approximately 2.8933. You will have to split the region into two pieces and set up an area integral on each piece.

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Problem 2.3 (10 points)**.** Find the volume of the solid obtained by rotating the region bounded by

$$y = \frac{1}{x^2 - 3x + 2}, \qquad y = 0, \qquad x = 3, \qquad \text{and} \qquad x = 4.$$

with respect to the x-axis.

Hint: Use the method of partial fraction decomposition.

Problem 2.4 (20 points)**.** Compute the arc length of the curve

$$y = \ln x$$

from $(1, 0)$ to $(e, 1)$.

Hints:

(1) It helps to make a preliminary substitution by setting $u = \frac{1}{x}$. What is du?
(2) Be sure to simplify the trigonometric expressions you obtain before you attempt to integrate them.
(3) You won't have to do integration by parts more than once.

Problem 2.5 (10 points)**.** Derive the formula for the surface area of a solid torus with the outer radius R and the inner radius r.

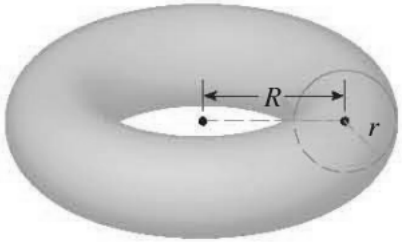

Image credit: James Stewart, *Essential Calculus: Early Transcendentals* (2e)

Hint: This is a textbook problem. See Section 7.5, Exercise 22.